

International Arms Trade:

A Dynamic Separable Network Model with Heterogeneity Components

Michael Lebacher[†], Paul W. Thurner[‡] and Göran Kauermann[†]

[†] Department of Statistics, Ludwig Maximilian University of Munich, michael.lebacher@stat.uni-muenchen.de

[‡] Geschwister Scholl Institute of Political Science, Ludwig Maximilian University of Munich, paul.thurner@gsi.uni-muenchen.de

[†] Department of Statistics, Ludwig Maximilian University of Munich, goeran.kauermann@stat.uni-muenchen.de



Abstract

We investigate data on international trade of major conventional weapons from 1950 to 2016. The general structure of the model is based on the separable temporal exponential random graph model (StERGM), but extends this model in two aspects. First, we allow for time-varying covariate effects and second, actor- and time-based heterogeneity is incorporated by smooth time-varying random effects. Furthermore, the random effects are deeper investigated using a functional principal component analysis. Our main findings are that arms trading is driven by network effects, notably reciprocity and triadic closure, but also by exogenous factors. The analysis of the country-specific random effects identifies countries that increased or decreased their relative importance in the arms trading network during the observation period.

The Model: StERGM + Generalized Additive Mixed Model

Goals

- Model the long-term development of arms trade
- Focus on the transition of the networks
- Account for time-related changes of the DGP
- Incorporate actor heterogeneity

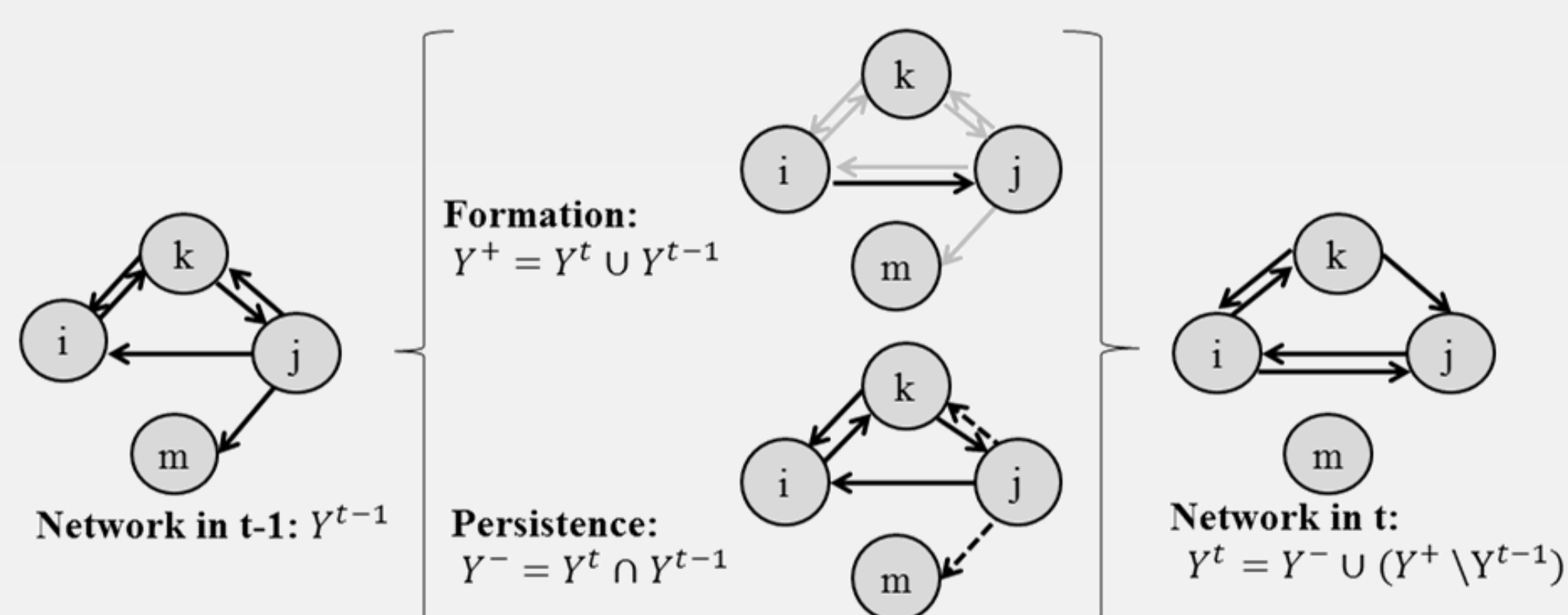
Solutions

- Separable temporal network model
- Varying coefficients and random effects

Formalizing the network

- Binary network in t : $Y^t = (A^t, E^t)$ with A^t being the actor set with n_t countries and $E^t = \{(i, j) : i, j \in A^t\}$ gives the set of directed edges.
- We set $Y_{ij}^t = 1$ if country i sends arms to j .
- Analysis is restricted to countries existing in t and $t-1$

The StERGM idea (see Krivitsky and Handcock, 2014)



The model is assumed to be separable given the previous network:

$$P(Y^t = y^t | Y^{t-1} = y^{t-1}; \theta) = P(Y^+ = y^+ | Y^{t-1} = y^{t-1}; \theta^+) P(Y^- = y^- | Y^{t-1} = y^{t-1}; \theta^-)$$

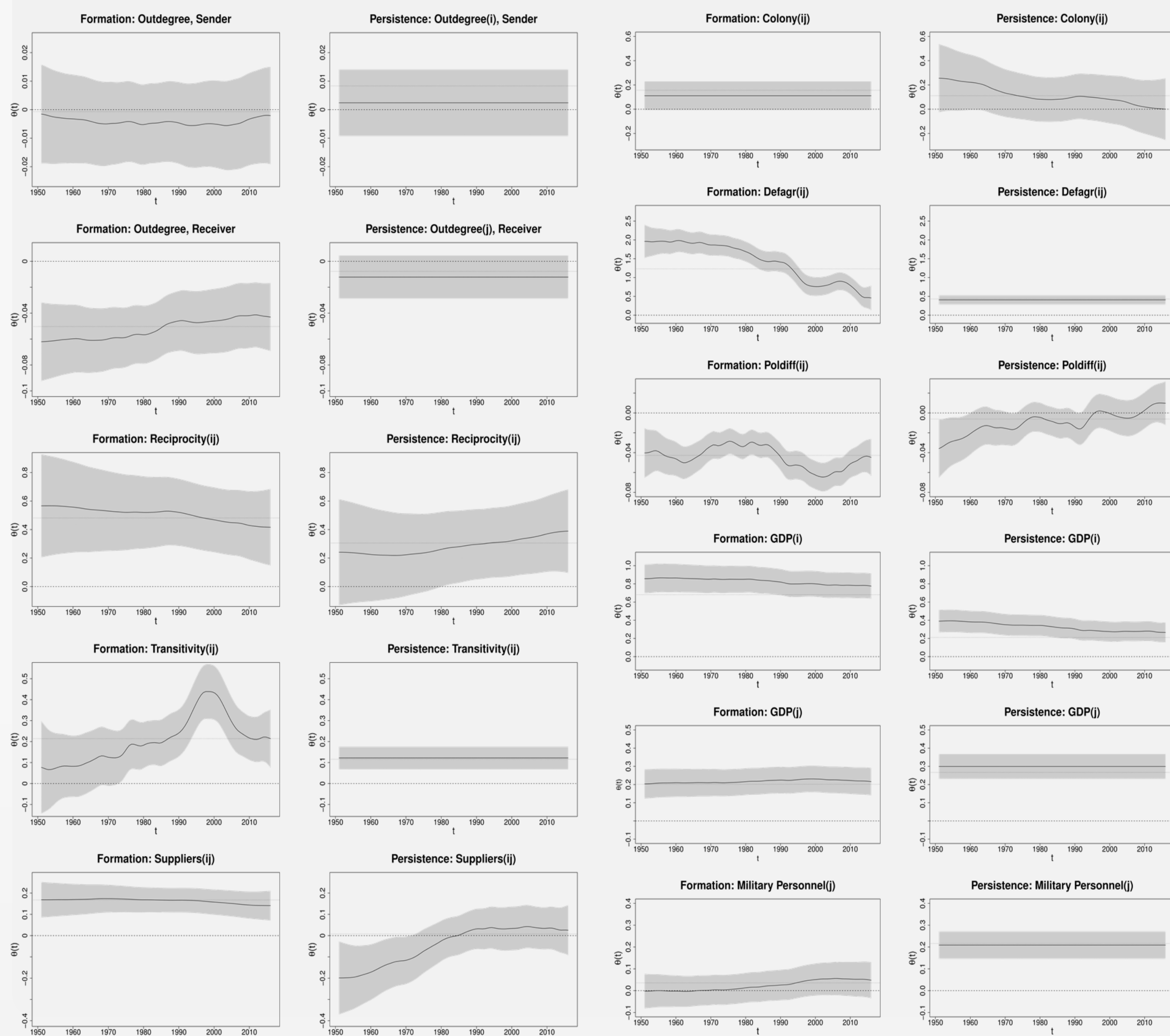
Binary network models are often modelled using ERGMs, but we follow Almquist and Butts (2014) and assume for the network statistics: $g(y^+, y^{t-1}) = \sum_{(i,j) \in E^t} y_{ij}^+ \tilde{g}_{ij}(y^{t-1})$.

$$\rightarrow \log \left\{ \frac{P(Y_{ij}^+ = 1 | Y_{ij}^- = y_{ij}^-, Y^{t-1} = y^{t-1}; \theta^+)}{P(Y_{ij}^+ = 0 | Y_{ij}^- = y_{ij}^-, Y^{t-1} = y^{t-1}; \theta^+)} \right\} = \log \left\{ \frac{P(Y_{ij}^+ = 1 | Y^{t-1} = y^{t-1}; \theta^+)}{P(Y_{ij}^+ = 0 | Y^{t-1} = y^{t-1}; \theta^+)} \right\} = \theta^+ \tilde{g}_{ij}(y^{t-1})$$

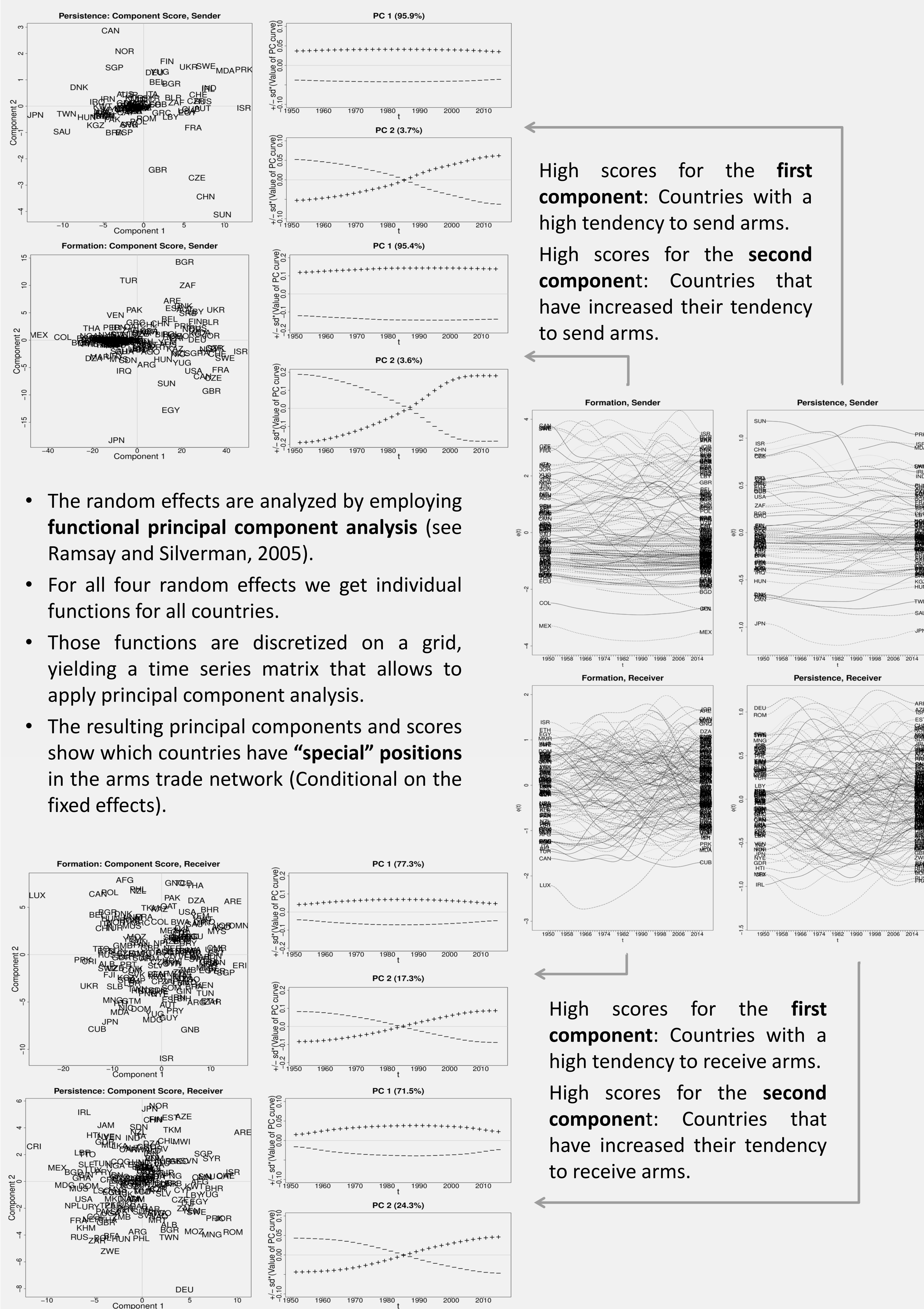
Model specification

- Replacing θ^+ by $\theta^+(t)$: Time-varying coefficients model (see Hastie and Tibshirani, 1993).
- Time-dependent random coefficients for the sender and receiver (see Durbán et al., 2005).

Results: Time-varying fixed effects



Results: Time-varying individual random effects



High scores for the **first component**: Countries with a high tendency to send arms.

High scores for the **second component**: Countries that have increased their tendency to send arms.

- The random effects are analyzed by employing **functional principal component analysis** (see Ramsay and Silverman, 2005).
- For all four random effects we get individual functions for all countries.
- Those functions are discretized on a grid, yielding a time series matrix that allows to apply principal component analysis.
- The resulting principal components and scores show which countries have **"special" positions** in the arms trade network (Conditional on the fixed effects).

High scores for the **first component**: Countries with a high tendency to receive arms.

High scores for the **second component**: Countries that have increased their tendency to receive arms.

Conclusion

The proposed model allows to study the process of formation and dissolution separately as well as the inclusion of time-varying coefficients random effects. We find network effects, especially for the triadic network statistics transitivity and shared suppliers as well as reciprocity. We also find interesting time variation in the coefficients on defence agreements and homophily, measured by the absolute difference of the polity score. For the economic quantities we have solid positive effects on the probability that a tie forms newly or persists. A careful analysis of the random effects exhibits a lot of variation among the countries as well as along the time dimension. The evaluation of the out-of-sample-fit confirms that the chosen model is a reasonable one.

References

- Almquist, Z. W. and C. T. Butts (2014): Logistic network regression for scalable analysis of networks with joint edge/vertex dynamics, *Sociological methodology* **44**, 273-321.
- Durbán, M., J. Harezlak, M. Wand, and R. Carroll (2005): Simple fitting of subject-specific curves for longitudinal data, *Statistics in Medicine* **24**, 1153-1167.
- Hastie, T. and R. Tibshirani (1993): Varying-coefficient models, *Journal of the Royal Statistical Society. Series B (Methodological)*, 757-796.
- Krivitsky P. N. and M. S. Handcock (2014): A separable model for dynamic networks, *Journal of the Royal Statistical Society. Series B (Methodological)*, 29-46.
- Lerner J., Indlekofer N., Nick B. and U. Brandes (2013): Conditional independence in dynamic networks, *Journal of Mathematical Psychology* **57**, 275-283.
- Ramsay, J. O. and B. W. Silverman (2005): *Functional data analysis*, Springer Science & Business Media.
- SIPRI (2017): Arms Transfers Database, <https://www.sipri.org/databases/armstransfers>.
- Wood, S. N. (2006): *Generalized additive models: an introduction with R*, CRC press.
- Wood, S. N., Y. Goude, and S. Shaw (2015): Generalized additive models for large data sets, *Journal of the Royal Statistical Society: Series C (Applied Statistics)*, **64**, 139-155.